Stable Coulomb bubbles ?

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Within the framework of the liquid drop model, the energy of a bubble in units of twice the surface energy of the equivalent sphere (constant volume) can be expressed as a function of the bubble monopole coordinate [1]. At zero pressure and angular momentum, the surface energy increases as a bubble develops from a sphere, but the Coulomb energy decreases as the charges are brought farther apart due to the bubble expansion. Therefore, an interplay between the Coulomb and surface energies may generate a minimum energy point along the monopole coordinate. The bubble minimum appears first at a value of the fissility parameter X = 2.022, and becomes the absolute minimum at X = 2.204.

A Coulomb bubble that is stable against monopole oscillations, may be subjected to higher order perturbations. The higher deformation modes of the bubble can be divided into two classes: the *radial modes* and the *crispation modes.* The deformations on the two surfaces are in phase with each other for a radial mode, and they are out of phase for a crispation mode.

The monopole oscillation obviously belongs to the class of radial modes. The lowest order crispation mode is the dipole mode which corresponds to a rigid displacement of the two spheres, one with respect to the other.

Unlike the dipole oscillation, higher multipole perturbations tend to increase the surface energy, and thus stabilize the unperturbed bubbles. This surface effect is the same for the radial and crispation modes, since the two modes differ only in the relative orientation of their surfaces. However, the Coulomb effect is drastically different for the two modes. The Coulomb perturbation energy is always negative for the radial mode, since the average distance between charges is increased slightly due to the perturbation. A similar effect of Coulomb destabilization is observed for the crispation mode in case of thick bubbles. In fact, the two modes are indistinguishable for a solid sphere. However, this destabilization effect becomes progressively weaker as the bubble expands. When a bubble is sufficiently thin, the Coulomb perturbation energy becomes positive, and stabilizes the crispation modes. This is because the Coulomb force tends to resist the attempt to concentrate the charge in "clumps" distributed on the surface of the thin bubble, as required by the higher order crispation modes. At the threshold fissility of X = 2.022, the value of X_{eff} lies just about at the n=4 line, indicating that the bubble is unstable up to the $n = 4$ mode. As more charge is brought into the bubble with increasing values of X, the Coulomb bubble expands and it becomes stable with respect to the $n = 4$ and even to the octupole mode ($n = 3$) at X=2.5. However, the Coulomb bubble is still unstable with respect to the quadrupole mode ($n = 2$). In fact, a further increase of X does not stabilize the quadrupole mode.

Yet, it may be possible to have a stable nuclear bubble. If the bubble is warm, it fills up with vapor arising from the fluid itself. The resulting pressure acts only upon the monopole mode, by displacing outwards the Coulomb minimum. The effect on the other radial modes is nil, since only changes in volume are relevant to pressure. Thus the bubble has become secularly stable with respect to all the modes.

However, when a bubble becomes rather thin, a possible demise of the bubble may be associated with the sheet instability. The sheet instability is a new kind of Rayleigh-like surface instability associated with the crispation modes. A nuclear sheet of any thickness tends to escape from the high surface energy by breaking up into a number of spherical fragments with less overall surface. However, any perturbation of finite wavelength increases the surface area, and consequently the energy of the sheet, independent of the sheet thickness. Clearly, this barrier prevents the sheet from reaching the more stable configurations. However, when a nuclear sheet becomes sufficiently thin, the two nuclear surfaces interact with each other. This proximity interaction may become sufficiently strong to overcome the sharp barrier and causes the sheet to puncture into numerous fragments.

Since a bubble, like a sheet, must rely on the proximity interaction to become unstable, it will retain its surface stability until the range of the surface-surface interaction is of the order of its thickness.

In conclusion, the depletion of charge in the central cavity of nuclear bubbles reduces the Coulomb energy significantly and thus stabilizes "Coulomb" bubbles against monopole oscillations. These Coulomb bubbles, however, are at least unstable to perturbation of the quadrupole radial mode. On the other hand, a sufficiently high temperature generates a vapor pressure in the central cavity which drives the bubble to a thinner configuration that is stable against all the radial modes. Finally, a thin Coulomb bubble behaves like a sheet, and becomes susceptible to a proximity surface instability via the crispation modes when its thickness is comparable to the range of the proximity interaction.

[1] L.G. Moretto, K. Tso and G.J. Wozniak, Phys. Rev. Lett. (accepted), LBNL-39678.